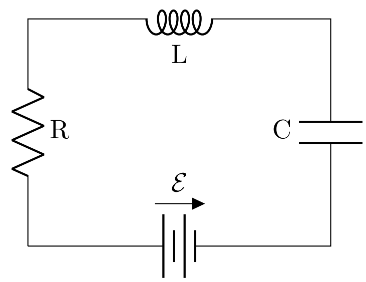
First we have the formulas for the ΔV through each component. Because the current around the entire circuit should be uniform, $\frac{dq}{dt}$ on the capacitor is equal to I, so q is $\int I\,dt$

$$\Delta V_C = \frac{1}{C}q = \frac{1}{C}\int I\,dt$$

$$\Delta V_R = IR$$

$$\Delta V_L = L\frac{dI}{dt}$$

This is kind of unrelated, but this setup really reminded me of a PID controller, with $P = R$, $I = \frac{1}{C}$, and $D = L$. I guess the setpoint would be related to the $\mathcal{E}$ of the battery, and the process variable would be I.



Let $\mathcal{E}$ be the voltage of the battery, R be the resistance of the resistor, C be the capacitance of the capacitor, and L be the inductance of the inductor. Using Kirchhoff's Law, we have

$$\begin{aligned}\mathcal{E} &= \Delta V_C + \Delta V_R + \Delta V_L \\ &= \frac{1}{C}\int I\,dt + RI + L\frac{dI}{dt} \\ &= \frac{1}{C}q + R\dot{q} + L\ddot{q}\end{aligned}$$

So this is a second-order differential equation. Though the best part is that it's linear, or almost so. Solving for $\ddot{q}$, we have

$$\frac{d^2q}{dt^2} = \frac{\mathcal{E}}{L} - \frac{1}{LC}q - \frac{R}{L}\dot{q}$$

and

$$\frac{dq}{dt} = \dot{q}$$

If $\frac{\mathcal{E}}{L} = 0$, then we can solve this using the method I learned about. Assuming that there is no battery (or at least $\mathcal{E} = 0$), we can write this differential equation as

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

We know the solutions to this equation will look like

$$q(t) = c_1 e^{\lambda_1 t} v_1 + c_2 e^{\lambda_2 t} v_2$$

So we can solve for the eigenvalues and eigenvectors and plug in

$$\begin{aligned} P_A(\lambda) &= (-\lambda)(-\frac{R}{L} - \lambda) + \frac{1}{LC} \\ &= \lambda^2 + \frac{R}{L}\lambda + \frac{1}{CL} \end{aligned}$$

That looks really ugly, so I don't want to do it out, but we can get a pretty good idea of what happens in certain scenarios.

If $R = 0$, then $P_A(\lambda) = \lambda^2 + \frac{1}{CL}$, so the eigenvalues will be $\pm\sqrt{\frac{1}{CL}}i$, which will lead to a solution that looks something like e^{kit}. So, like in our solutions to SHM problems, there is some oscillation going on. So the charge on the capacitor will oscillate up and down provided $R = 0$, which implies the current oscillates too.

If instead $R \neq 0$, it will probably be similar to the previous case but instead the oscillation will die down over time because energy should be dissipated in the resistor, like if we were looking at a spring with drag on it.

These solutions are all for the case without a battery, so I guess you would have it connected via a switch to some other circuit with a battery, and once a current is generated, you disconnect the switch to isolate this setup.

If there is a battery, then there is some shift in the equations so that they are no longer linear. I'm not sure what this means, but it could oscillate around a different charge level? (Everything in this example oscillates around or collapses towards $q = 0$) Or maybe it doesn't oscillate at all and just stabilizes around some fixed charge value.